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NUMERICAL MODELING OF MECHANICAL, RHEOLOGICAL, AND STRENGTH PROPERTIES OF DISPERSE SYSTEMS

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The method of structural elements is considered. With its help, a modeling of the stressed-strained state of easily deformable disperse systems is performed. On the basis of computational experiments on triaxial compression, the conditions of failure of such materials are determined and the cracking in them is investigated.

For solving problems of heat and mass transfer with structure transformation, we have devised a numerical method based on the analysis of structural elements. As was shown earlier [1-3], problems of heat and mass transfer in the case of cracking and failure of material can be successfully solved with the use of discrete structural elements. The distinctive features of this method lie in the fact that, in the case of structural transformations, it enables one, along with energy and mass conservation, to allow adequately for the change in the characteristics of heat and mass transfer. This provides the possibility of solving problems on drying of easily deformable materials and their thermoelasticity. In this case, the form and structure of the material are uniquely determined by the moisture-content and temperature fields. When this method is used for modeling of complex rheological processes, major attention should be focused on the mechanical motion of structural elements.

In the present work, additional possibilities of the method are demonstrated and, on its basis, problems of numerical modeling of the mechanical, rheological, and strength properties of natural disperse systems are considered. We describe below certain stages and algorithms of calculation of the processes of structure transformation and soil deformation.

The basic assumption used in solving problems of mechanical motion is that the total force acting on a structural element tends to displace it to the position where the value of this force approaches zero. In the general case, the structural element is acted upon by forces from other elements, which are due to the deformation of linear links (structural forces), the change in the volume of the element (volume forces), and the difference between the velocities of neighboring elements (viscous forces). In addition, inertial forces can arise due to the change in the velocity of motion of the element. In investigating the deformation processes in soils and grounds, we can restrict our consideration to the first two forces.

With the foregoing, the displacement of a structural element in each time step is determined by the equation

$$\left(F + \frac{\partial F}{\partial r}\Delta r\right)_{\vec{z}} = 0 . \tag{1}$$

Using formula (1), we calculate the displacement of the element and its velocity by the formulas

$$\Delta r = -\frac{F}{\frac{\partial F}{\partial r}} , \tag{2}$$

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$$v = \frac{\Delta r}{\Delta \tau} . \qquad (3)$$

Calculation of the displacements and the velocities is performed element by element on each time layer and in two steps independently of the positions of the elements, at first, in order of increasing numbers of the elements, and then in the reverse order. In the first case, for neighboring elements that have a number larger than the number of the considered element we take into account the displacements by a certain assumed value $v_i \Delta\tau$, where v_i is the velocity of the corresponding element calculated on the previous time layer. In the second case, conversely, one treats in the same manner elements whose number is smaller than the number of the considered element. This procedure makes it possible to transfer more adequately mechanical disturbances from one region of the material to another.

We present the algorithm of calculation of the quantity $\partial F/\partial r$:

1) calculation of the force at the point (x_1, y_1);

2) determination of the point with coordinates (x_2, y_2) at a distance Δr_1 along the force direction;

3) calculation of the force acting on the element at the point (x_2, y_2);

4) if the force direction at the point (x_2, y_2) differs from the force direction at the point (x_1, y_1), it is necessary to decrease Δr_1 and return to item 2;

5) calculation of the modulus of the difference of the forces at the points (x_1, y_1) and (x_2, y_2) and division by Δr_1.

Knowing the force at the point (x_1, y_1) and the derivative $\partial F/\partial r$, we calculate the displacement Δr by formula (2). It is plotted along the force direction, which makes it possible to find new coordinates of the element (x_3, y_3). The new velocity is calculated by formula (3).

We considered the force that acts on the elements and represents the total resultant force from the action of a number of forces. Their nature and number depend on the environmental conditions and the rheology of the system. It is known that the rheological properties of the system can be described by the combination of three fundamental properties: elasticity, viscosity, and plasticity. Their possible realization in structural elements is presented in [1-3]. We dwell only on the plastic properties. They can be modeled through both the plastic properties of linear links and the plasticity of the structural-element volume. We consider plastic deformations of the links. In modeling a plastic flow through linear links we proceed from the assumption that the mean hydrostatic pressure on the element and its mass must remain unchanged. If the density of the body is constant, the mass constancy can be replaced by the constancy of the initial volume of the system. For simplicity, we will consider the procedure of realization of plastic deformations for the two-dimensional case. In the process of heat and mass transfer, certain stresses occur on the links of the element. In this case we can calculate the average stress on the element by the formula

$$P = -\frac{1}{N} E \sum_{i=1}^{N} \frac{\Delta l_i}{l_i} . \qquad (4)$$

In this case, plasticity manifests itself as follows: the initial length of the links on which the stress is larger than the average stress decreases, and the initial length of the links on which the stress is smaller than the average stress increases. If the stresses on the links are equal in value and sign, the initial lengths of the links remain unchanged. This situation is realized in the case of triaxial compression or expansion. We write the general algorithm of realization of plastic deformations on the links:

1) calculation of the average stress on the element;

2) calculation of the change in the initial lengths of the links of the element relative to the average stress: $\Delta l_{0i} = \text{koef} \left(\frac{P}{E} l_i + \Delta l_i \right)$;

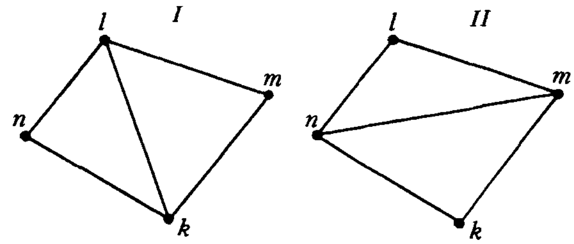

Fig. 1. Modeling of the replacement of links.

3) determination of the change in the initial lengths of the links that are due to the conservation of the initial area of the element; in this case the formula $\Delta l_{0bi} = \frac{\sum_j \Delta l_{0j} l_{0j}}{\sum_j l_j^2} l_{0i}$ is used;

4) calculation of the final change in the initial lengths of the links $\Delta l_{0f} = \Delta l_{0f} + \Delta l_{0bi}$;

5) calculation of the new values of the initial lengths of the links.

In this case, averaging of the stressed state on the links takes place, tangent stresses approach zero, and irreversible plastic deformations arise in the material.

Now we dwell on the plastic deformations related to a change in the volume. In this case, besides the element in itself, it is necessary to consider its neighbors. Using the values of the hydrostatic pressures on the element and on its neighbors, we calculate the mean hydrostatic pressure. Then we use it to find the new value of the initial volume of the element. In this case we will assume that the sum of the initial volumes of the elements and of its neighbors remains unchanged prior to plastic transformations and after them. Therefore the change in the initial volume of the element should be compensated for by the change in the initial volumes of its neighbors so that their total volume remains unchanged. We present the general algorithm of the above discussion:

1) calculation of the mean hydrostatic pressure on the element by the formula $P_{hi} = \frac{1}{3}\left(2\frac{\sum_{k=1}^{M} P_k}{M} + P_i\right)$;

2) determination of the new value of the initial volume of the element $V_{0ni} = V_i \frac{P_{hi} + E_v}{E_v}$;

3) calculation of the difference between the previous and new values of the initial volume $\Delta V_{0i} = V_{0bi} - V_{0ni}$;

4) to extend this difference to the neighboring elements, it is necessary to calculate the increase in the initial volumes of the neighboring elements $\Delta V_k = \Delta V_{0i}(P_k - P_i)/\sum_k (P_k - P_i)$;

5) determination of the new values of the initial volumes of the neighboring elements $V_{0nk} = V_{0bk} + \Delta V_k$, where V_{0bk} is the previous initial volume of the neighboring element.

When this procedure is performed, the elements exchange initial volumes, relaxation of the hydrostatic pressure takes place, and, as a consequence, plastic deformations occur.

Plastic deformations also arise as a result of the replacement of links between the elements. This procedure is intended to provide the possibility of motion of the elements for long distances relative to each other. The element can be displaced from one end of the body to another, and its neighbors can be completely replaced. Such a free displacement of some elements relative to others, besides plastic deformations, makes it possible to model a viscous flow and the fluidity of the material.

We consider the procedure of replacement of links. It is based on the application of the notion of weight. Let us assume that we have four elements n, m, l, and k. In this case, the variants of links between the elements shown in Fig. 1 are possible. Let us assume, for definiteness, that prior to the procedure of replacement of links, variant I was realized. Let the weight of state I be $g1$ and the weight of state II be $g2$. If weight

$g1$ is larger than weight $g2$, state I is realized. If weight $g2$ is larger than weight $g1$, state II is realized. If the weights are equal, the initial variant I remains. In this case, the weight is determined by a large number of conditions. Thus, it depends on:

1) the value of the stress on the links; if it exceeds the admissible value, a link is broken or is replaced; a concrete realization depends on the specific character of the problem;

2) the length of the links; preference is given to a shorter link;

3) the number of links in the element;

4) the quality of the links; the body can represent a composite material consisting of solid particles possessing the properties of air, water, and other inclusions; it is apparent that in this case the properties of the elements and, respectively, of the links are different;

5) the value of the random quantity; this is necessary to impart stochastic properties to the system;

6) the value of the angle between the links.

This list can be supplemented depending on the specific properties of the problem solved. Giving the necessary weights to different conditions, we manage to control the possibility of replacing the links in the necessary direction. Consequently, we can also exercise control over a number of rheological properties of the system.

When a link is broken, heat-and-mass-exchange and mechanical interactions between elements disappear at the site of the break. However, the cracks actually formed are filled with a different substance. In particular, it can be air or water. Therefore, to allow more correctly for the heat and mass exchange of the system with the environment, where the cracks arose it is necessary to create elements with new heat-and-mass-exchange and rheological properties that correspond to the environment. At first, at the site of the broken link a new link with properties of the substance filling the crack is formed. As the crack increases further, at the instant its dimensions satisfy the condition of formation of an element, a quadrangular element with properties of the environment is already formed. As the crack increases, new elements can be formed on each link of the element. In this way the crack is gradually filled with new elements. This allows the disperse-medium elements to retain the capacity for heat and mass exchange in the case of cracking. A different situation is also possible. At the final stage of the process of heat and mass transfer the transfer gradients in the body decrease; as a consequence, the extending stresses decrease, which can lead to a decrease in the dimensions of the cracks and even to their complete collapse. Therefore, the possibility of removal of environmental elements has been realized. In this case, the size of the element is examined; if it is less than a certain value, the element is transformed to a quadrangle through the replacement of the links with other elements. Thereafter we can remove it, leaving only one link. In this way all the elements can be removed until the complete collapse of the cracks.

We write the general algorithm of formation of elements:

1) formation of a new link with properties of the environment;

2) formation of a quadrangular element with new properties when the condition of breaking of the new link is satisfied;

3) as the crack increases, items 1 and 2 are repeated as many times as necessary.

The algorithm of removal of elements:

1) calculation of the size of the element;

2) if it is less than a certain value prescribed by the data given, the number of links in the element is determined;

3) if this number is larger than four, it is increased to the necessary value with the procedure of replacement of links;

4) removal of the element;

5) if the crack decreases further, this procedure is repeated as many times as necessary.

It should be noted that the most complete and sequential description of the method of structural elements and the main results obtained on its basis are presented in [4].

Using the above-described method, we consider the mechanical behavior and the rheological properties of structured easily deformable rocks. Actual bodies are three-dimensional. One-dimensional or two-dimensional models can be inadequate to describe them. This primarily concerns the description of the mechanical motion of materials, which forces us to model three dimensions. In investigating the complex stressed state, cylindrically shaped samples are most often used, because of the fact that external stresses on them are simpler to prescribe. The cylindrical figure can be represented as the figure of rotation with the use of a two-dimensional rectangle. Since the cylindrical coordinate system is used, it is necessary to somewhat change the applied formulas. This problem differs from the plane problem by the existence of a force that is caused by the deformation of the circle of rotation. This force is directed along the radius to the center of the cylinder. The formulas were derived with regard for the features of the cylindrical coordinate system. The expression for the force that is caused by the deformation of the linear link between the i-th and k-th elements takes on the form

$$F_{ik} = \frac{\pi}{2} E\,(l_{ik} - l_{0ik}) \left(\tan\frac{\pi}{N_i} + \tan\frac{\pi}{N_k} \right) (R_i + R_k) ,$$

where l_{0ik} is the initial length of the link between the i-th and k-th elements.

Besides this force, the force that is related to the deformation of the circle of rotation acts in the considered plane:

$$F_i = 2\pi E \frac{R_i - R_{0i}}{R_{0i}} S_{\text{el}} .$$

The force that is due to the difference between the hydrostatic pressures on the elements is calculated by the formula

$$F_{ik} = \frac{\pi}{2} (P_{vi} - P_{vk}) \left(\tan\frac{\pi}{N_i} + \tan\frac{\pi}{N_k} \right) (R_i + R_k)\, l_{ik} .$$

The force acting from the environment at the boundary has the form

$$F_{ik} = \frac{\pi}{4} (P_{vi} - P_{\text{vext}}) (3R_i + R_k)\, l_{ik} .$$

The hydrostatic pressure P_{vi} is calculated as

$$P_{vi} = E_v \frac{V_{0i} - V_i}{V_i} . \tag{5}$$

The expression for the average stress on the links has the form

$$P_a = \frac{1}{3} \left(2E \frac{\sum\limits_k \frac{\Delta l_k}{l_k}}{N} + E \frac{R_i - R_{0i}}{R_{0i}} \right) . \tag{6}$$

Relaxation of the stresses on the links between the i-th and k-th elements is performed by the following formulas, written in order of their application:

$$\Delta l_{0ik} = a \left(\frac{\sum\limits_k (l_{ik} - l_{0ik})}{N} - (l_{ik} - l_{0ik}) \right), \quad \Delta R_{0i} = a_1 \left(\frac{\frac{l_{ik}}{l_{0ik}}}{\frac{R_i}{R_{0i}}} - 1 \right) R_{0i} ,$$

$$\Delta l_{0ik} = -0.25m \frac{\Delta R_{0i}}{R_{0i}} l_{0ik}.$$

When these relaxation formulas are used, the situation where the initial length of the link l_{0ik} approaches zero may occur. In view of the physical fact that actual disperse particles can change their sizes only in a certain range, the change in l_{0ik} must be limited. Let $l_{0\max}$ and $l_{0\min}$ be respectively the maximum and minimum values of the initial length of the link l_{0ik}. In this case, the range of variation of the relative value of the initial length of the link can be represented as $\varepsilon_0 = (l_{0\max} - l_{0\min})/p_{0a}$, where p_{0a} is the mean value of the initial length. Changing ε_0, we can control the plastic properties of the modeled material. In this case, it is well to bear in mind that $\varepsilon_0 = 0$ does not mean that the relaxation of the stresses on the elements is absent. In this case, $p_{0a} = l_{0\max} = l_{0\min}$ and all the links tend to p_{0a} as much as possible.

Developing the method of modeling of the processes of deformation and transformation of the structure of three-dimensional axially symmetric bodies makes it possible to simulate standard mechanical testing of samples, which allows one to prescribe adequately the parameters of models that correspond to concrete grounds and rocks. We consider the sample in the form of a cylinder. The sizes of the sample are chosen in much the same manner as in laboratory experiments: the height of the sample $H = 1.5D$, where D is its diameter. In view of the fact that the symmetry is cylindrical, we consider half of the cross section of the sample along the vertical axis. In line with this, the area of the considered cross section is broken into $NG \times NV$, where $NG = 20$ and $NV = 60$. To carry out a computational experiment, we have performed two types of compression: triaxial compression and compression with the possibility of side expansion. In this case, the ratio between the volume and linear moduli of elasticity $\chi = E_v/E$, the magnitude of ε_0, and the external pressure applied to the lateral surface $P_{\text{lat.ext}}$ are varied. In the calculations, the value of the linear modulus of elasticity was taken to be $E = 1000$ kPa.

For testing the problems and determining the physicomechanical characteristics of the models of grounds, we compared the parameters of the investigated medium as a continuous body with the parameters of structural elements into which this body is broken. Triaxial compression makes it possible to determine the rheological properties of the sample from the rheological parameters of the structural elements. To do this, the sample was compressed without the possibility of side expansion with a relative deformation of $\varepsilon = 0.1$. Then we determined the pressures P_e and P_{lat} on the end and lateral surfaces, respectively. Thereafter we calculated the compression modulus of elasticity E_{com}, the standard modulus of elasticity E_s, and the Poisson coefficient ν of the entire body by the following formulas: $E_{\text{com}} = P_e\varepsilon$, $\xi = P_{\text{lat}}/P_e$; $\nu = \xi/(1+\xi)$; $E_s = E_{\text{com}}(1+\nu)(1-2\nu)/(1-\nu)$. The corresponding values of these quantities as functions of χ and ε_0 are presented in Table 1. It is seen from the table that, varying the parameters χ and ε_0, we can model the rheological properties of the body in a wide range that corresponds to the range of variation of the properties of actual easily deformable natural disperse systems. Thus, with increase in χ, which corresponds to an increase in the modulus E_v at a constant E, the elastic constants E_{com}, E_s, and ν increase. In this case, the contribution of the forces caused by a change in the volume increases. Selecting the parameters E_v and E, we can obtain the necessary values of E_{com}, E_s, and ν. Finally, we manage to model the elastic behavior of the entire sample. The model of an elastic-plastic body has also been realized. The existence of the relaxation procedure gives an additional redistribution of stresses on the links. With it, it is also possible to model deformations that are not restored after the removal of the load, which enables us to speak of plastic deformations. The loads are redistributed so that the stresses on the links become equal in different directions. The transfer of a portion of the stress in the other direction can be characterized by the quantity ε_0. It is seen from the table that E_v and E decrease with increase in ε_0, which is due to the relaxation of the load P_e at the ends. Simultaneously with this, the Poisson coefficient characterizing the transfer of the load in the perpendicular direction increases. Thus, by selecting the parameters E_v, E, and ε_0, we can model the necessary properties of the entire sample in both the elastic and elastic-plastic regions.

We consider the triaxial compression of a sample with the possibility of side expansion. For elastic-plastic samples we estimated the rate of stress relaxation. Analysis of the relaxation made it possible to select

TABLE 1. Dependence of the Compression Modulus of Elasticity E_{com} (kPa), Modulus of Elasticity E_s (kPa), and Poisson Coefficient ν on ε_0 and χ.

χ	$\varepsilon_0 = 0.2$	$\varepsilon_0 = 0.1$	$\varepsilon_0 = 0.05$	$\varepsilon_0 = 0$	Without relaxation
			E_{com}		
0.03	419	517	572	626	764
0.1	491	589	645	699	842
0.3	697	794	849	904	1053
0.5	898	997	1051	1107	1259
			E_s		
0.03	74	187	277	366	703
0.1	80	193	285	375	734
0.3	88	204	297	391	785
0.5	88	211	301	401	814
			ν		
0.03	0.468	0.427	0.395	0.363	0.181
0.1	0.471	0.436	0.407	0.379	0.223
0.3	0.478	0.451	0.430	0.409	0.298
0.5	0.483	0.461	0.445	0.427	0.341

the time step and the rate of compression of the samples which allows plastic deformations to be realized and leaves the result obtained unaffected.

The stressed state at a concrete point of a sample consisting of discrete structural elements can be characterized by the average stress on the links of the element P_a, the stress related to the deformation of the circle of rotation P_r, and the hydrostatic pressure P_v. Knowing the deformation of individual links and the modulus of elasticity of the links, we can calculate the stress-tensor components that correspond to the characteristics of the stressed state of a continuous medium by the formulas

$$P_{xx} = -E \frac{\sum_k (\Delta l_k / l_k) \cos^2 \alpha_k}{\sum_k \cos^2 \alpha_k}, \tag{7}$$

$$P_{yy} = -E \frac{\sum_k (\Delta l_k / l_k) \sin^2 \alpha_k}{\sum_k \sin^2 \alpha_k}, \tag{8}$$

$$P_{xy} = -E \frac{\sum_k (\Delta l_k / l_k) \cos \alpha_k \sin \alpha_k}{\sum_k \sin^2 \alpha_k}. \tag{9}$$

The component P_{xx} acts in the surface element perpendicular to the X axis on the direction of this axis. The component P_{yy} acts on the surface element parallel to the X axis in the direction of the Y axis. P_{xy} is applied to the same surface element to which P_{yy} is applied, but it is directed along the X axis. We can also obtain the values of the stress components acting on other surface elements inclined to the X axis at any angle ψ. To do this, in place of the angle α_k, it is necessary to take the angle $\alpha_k + \psi$. In this case, P_{yy} is directed perpendicularly to the surface element inclined at an angle ψ to the X-axis. The component P_{xx} acts on the

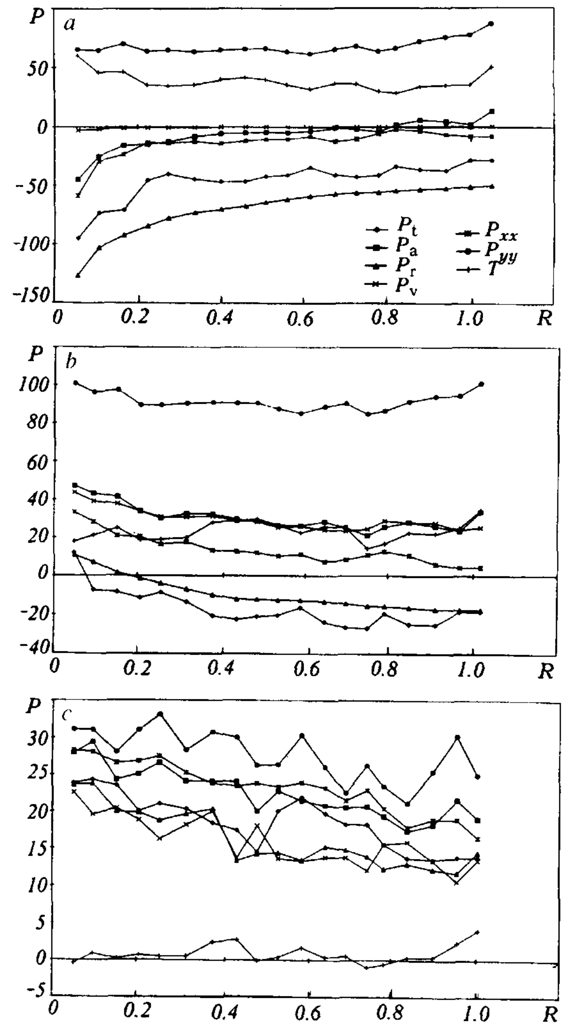

Fig. 2. Radial distribution of stresses in a cylindrical sample in testing (a) and triaxial compression of elastic (b) and elastic-plastic (c) samples. P, kPa; R, dimensionless radius.

element positioned perpendicularly to the considered one along the direction determined by the angle ψ. Correspondingly, the component P_{xy} is a tangent component acting on the given surface element.

It should be noted that the characteristics calculated by formulas (7)-(9) are static in character. Therefore, to describe a local stressed state, it is desirable to average these characteristics over a group of structural elements, for example, over the considered element and over its neighbors.

Let us analyze the test problem on the distribution of stresses in a sample. To do this, we compress it in the direction of the Y axis and extend along the X axis. In this case, the deformation must be of such a magnitude that the volume of the system remains unchanged. Figure 2a shows radial distributions of the stresses described above. The calculations have shown that P_{xy} has a maximum value on the surface elements positioned at angles $\pi/4$, $3\pi/4$, $-\pi/4$, and $-3\pi/4$ to the X axis. We denote the component P_{xy} acting at an angle $-\pi/4$ as T. Then, it is seen from Fig. 2a that the components P_{xy}, P_{yy}, and T are related by the equation

$$T = (P_{yy} - P_{xx})/2\,. \tag{10}$$

This conforms with the general theory of the state of plane stress. It is seen from Fig. 2a that the hydrostatic pressure is absent. Practically throughout the body, except for the center, the average stress on the links P_a

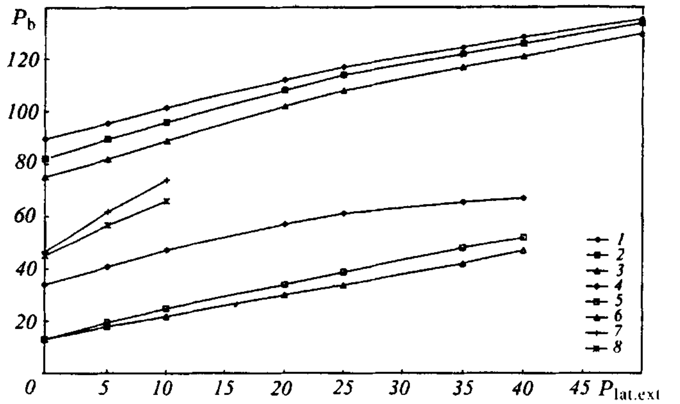

Fig. 3. Dependence of the breaking load on the side pressure: for elastic samples, curves 1-3 [1) χ = 0.03; 2) 0.3; 3) 0.5); for elastic-plastic samples, curves 4-6 [4) χ = 0.03; 5) 0.3; 6) 0.5]; for samples with different strength properties of the links, curves 7 and 8. P_b, $P_{lat.ext}$, kPa.

approximates zero. Only at the axis of the cylinder does P_a have a certain negative value, because of the dependence of the mean pressure on the deformation of the circle of rotation by formula (6). The stresses P_{link} and P_r behave analogously, and P_r is related to the change in the radius R_i of the element. The distribution of P_r is determined by the loads applied to both the end and lateral surfaces. The links arranged parallel to the X axis are in the extended state, which corresponds to the shape of the curve for P_{link}. The increase in P_{link} occurring when going to the axis is due to the pressures applied to the end surfaces.

We perform a computational experiment for two bodies – an elastic and an elastic-plastic body with ε_0 = 0.2. We vary both χ and the external pressure $P_{lat.ext}$ applied to the lateral surface of the body. Figure 2b and c shows the distributions of stresses in the elastic and elastic-plastic samples in the case of triaxial compression at $P_{lat.ext}$ = 0. For the elastic sample, the component P_{yy} takes on the maximum value. Taking into account fluctuations that are due to the stochastic character of the properties of the elements, it may be assumed that the values of P_{yy} are constant. Near a certain mean value, all the considered stresses experience fluctuations. The component P_{xx} is much smaller than P_{yy}, which indicates a poor transfer of stresses at small values of χ. With increase in χ, P_{xx} also increases. Since the external pressure is absent on the lateral surface, $P_{lat.ext}$ = 0, P_{xx} decreases with increase in the radius. On the average, relation (10) is fulfilled for T. The mean pressure on the links and the hydrostatic pressure are identical in behavior. They decrease slowly toward the side boundary in much the same manner as the component P_{xx} decreases. The stress P_r takes on a negative value and increases in modulus as the lateral surface is approached. A similar behavior is shown for P_{link}. This distribution of P_r is due to the increase in the radius of the elements, with the result that the body takes the shape of a barrel. The shape of the curve of P_{link} is explained by the features of the distribution of stresses on the links. The forces acting on the links that are arranged parallel to the X axis tend to extend them. At the same time, the links arranged along the Y axis are compressed. The distribution obtained for the elastic-plastic body is different in character and is determined by the relaxation of the stresses. As expected, this leads to the equalization of the stresses in different directions, as can be seen from Fig. 2c. The component T, on the average, is zero in accordance with (10). Comparison of these plots shows that the systems with relaxation and the systems without relaxation are different in mechanical behavior.

From the results of triaxial experiments we can also determine the conditions of failure of the sample. At first, the sample was compressed by the confining pressure till the state of equilibrium was established. Then the load that causes a deformation proceeding with a constant rate to failure was applied to its ends. In this case, we noted the maximum pressure on the end surfaces of the sample. In Fig. 3, the results of the computational experiment for the elastic and elastic-plastic bodies at different values of the parameter χ, where P_b is the breaking load, are respectively presented. Figure 3 also shows the curves of failure of the sample at

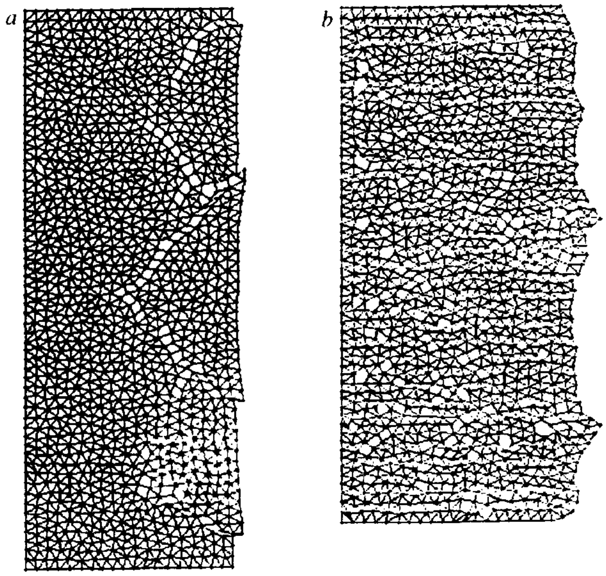

Fig. 4. Typical patterns of failure of elastic (a) and elastic-plastic (b) samples in triaxial compression.

different strength properties of the links. Curve 7 shows the linear dependence of the link strength on the pressure P_a on the element; curve 8 corresponds to the case of a constant strength of the link. Based on the failure curves, we can calculate the strength characteristics: internal-friction angle φ and bond c_0. It is significant that, for curve 7, the internal-friction angle is $\varphi = 27.8^{\circ}$ and the bond is $c_0 = 14$ kPa, and for curve 8, $\varphi = 20.8^{\circ}$ and $c_0 = 15.5$ kPa. These indices of strength correspond to peat systems [5]. Changing the parameters of the structural elements, one can select the strength indices that correspond to concrete dispersed materials.

In accordance with the physical, physicomechanical, and physicochemical properties of natural disperse systems, the results obtained can be interpreted as follows. The structural links reflect the mechanical properties of fibers that thread the entire body and interweave, forming the skeleton, and also contacts between the particles of the coarse fraction of the material. The volume modulus of elasticity E_v describes the properties of the moist nonstructured fine fraction that fills the spaces formed by the skeleton and larger particles. It is seen from Fig. 3 that the smaller the contribution of this fraction, the stronger the body. Comparison of curves 1-3 and 4-6 shows that the strength of the system with no relaxation is higher than the strength of the system with the possibility of relaxation. This can be explained by the higher mobility of the skeleton due to coagulation links and by the smaller contribution of the fibers to the breakage of the entire system.

The computational experiments have shown that elastic and elastic-plastic materials behave differently in cracking. Typical patterns of failure of elastic and elastic-plastic bodies are presented in Fig. 4a and b, respectively. In the elastic bodies, the process of cracking to the point of failure takes a shorter time interval. At the initial period, links break at one site, forming a crack that propagates in a certain direction. In this case, the pressure at the ends of the sample reaches the maximum at the initial stage of failure and then monotonically decreases to the complete failure of the sample.

The process of failure of the elastic-plastic samples takes a longer time interval. At the initial period, cracks are formed in a random manner throughout the sample. The pressure at the sample ends declines at the instant the links break. However, it increases again after a certain time interval to the next breaking of the links. Consequently, the pressure fluctuates with time about a certain mean value. In this case, three variants can be realized: the mean pressure increases, decreases, or remains constant. This situation exists up to a certain instant after which the pressure ceases to fluctuate and decreases monotonically, whereupon the sample fails.

The method of structural elements allowed us to describe the change in the shape of the sample and the transformation of its structure caused by the cracking and repacking of the elements in the volume, to estimate the stressed-strained state of the system, and to relate the rheological characteristics of the structural elements to the rheological properties of the entire body.

Thus, this method makes it possible to calculate all the basic parameters of the strain stressed state at each point of the material and to allow for the influence of mechanical stresses on the moisture distribution. This offers a means of solving such problems as filtration consolidation of grounds and dehydration of rocks with the help of mechanical loads and the problems of stability of slopes and modeling of landslips. Assigning the properties of different components of a multicomponent system to the structural elements, one can calculate the physicomechanical properties of this system with the help of the method proposed.

NOTATION

F, force acting on the element, N; r, radius vector of the element position, m; $\vec{z}$, unit vector of the force direction; Δr, displacement of the element, m; v, velocity of the element, m/sec; Δl_i, nonadmissible shrinkage, m; l_i, length of the link between the elements, m; l_{0i}, initial length of the link between the elements, m; koef, coefficient less than unity; N, number of links in the element; V_{0i}, initial volume of the element, m^3; V_i, volume of the element, m^3; E, modulus of linear deformation of the link, Pa; E_v, volume modulus of elasticity of the element, Pa; m, number of links participating in relaxation; P_i, pressure on the i-th element, Pa; P_k, pressure on the neighboring k-th element, Pa; l_{ik}, length of the link between the i-th and k-th elements, m; R_i and R_k, radii of the corresponding i-th and k-th elements, m; R_{0i}, radius of the element in the case of complete absence of moisture in the body, m; S_{el}, area of the element, m^2; Δl_{0ik} and ΔR_{0i}, increments in the initial length of the link and the initial radius of the element, respectively, m; ε_0, range of variation of the relative value of the initial length of the link; α_k, angle between the link and the X axis, rad; ψ, angle of slope of the surface element to the X axis, rad; a, a_1, coefficients less than unity; NG, number of elements along the horizontal; NV, number of elements along the vertical; P_e and P_{lat}, pressures on the end and lateral surfaces of the sample, respectively, Pa; E_{com}, compression modulus of elasticity, Pa; E_s, standard modulus, Pa; ν, Poisson coefficient; P_a, average stress on the links of the element, Pa; P_r, stress caused by the deformation of the circle of rotation, Pa; P_v, hydrostatic volume pressure, Pa; P_{link}, stress on the link, Pa; P_{xx}, P_{xy}, and P_{yy}, calculated components of the stress tensor, Pa; $P_{lat.ext}$, lateral external pressure, Pa; P_b, breaking load, Pa; φ, angle of internal friction; c_0, bond, Pa. Subscripts: el, element; e, end; lat, lateral; com, compression; ext, external; i, j, and k, numbers of elements; x, spatial coordinate along the X axis; y, spatial coordinate along the Y axis; s, standard; b, breaking; a, average; r, rotation; v, volume; link, link, f, final; b, previous; n, new.

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